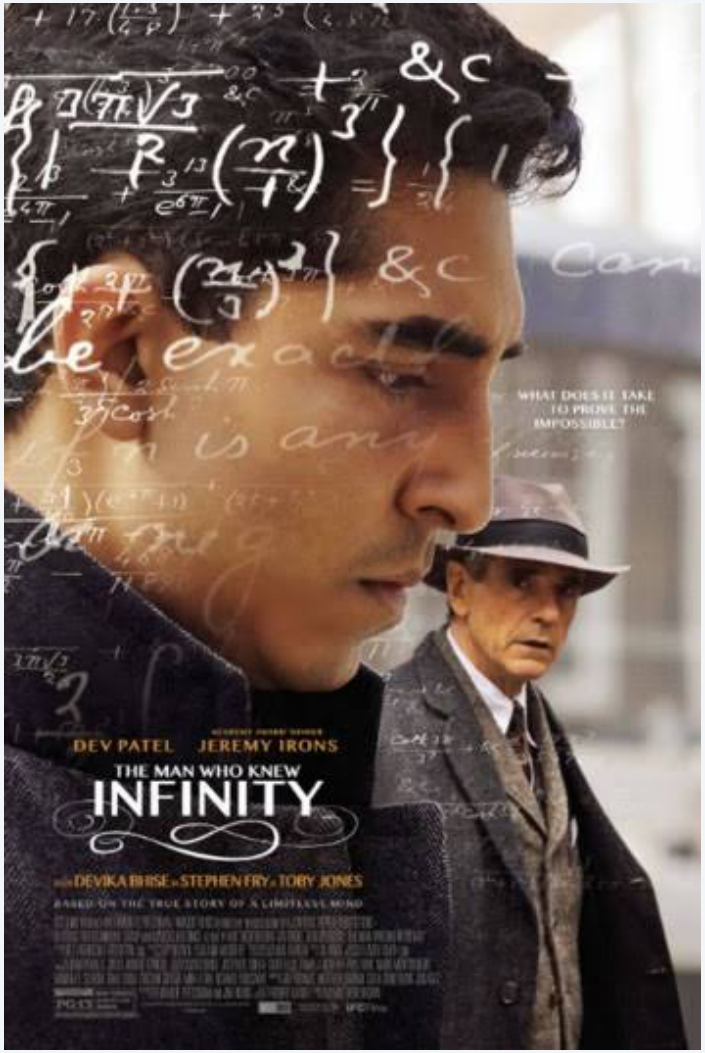
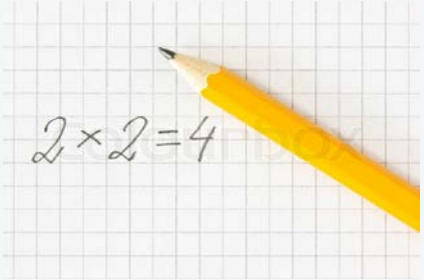


"An equation means nothing to me unless it expresses a thought of God."

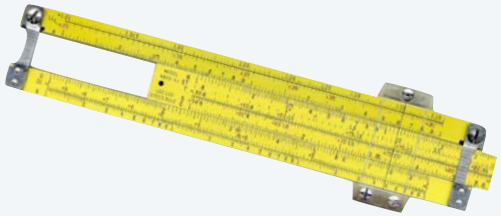
– Srinivasa Ramanujan

$$Q(N) \equiv \sum_{1 \le k \le N} \frac{N!}{(N-k)!N^k} = \sqrt{\frac{\pi N}{2}} + O(1)$$


BASED ON THE UNTOLD TRUE STORY
MEET THE WOMEN YOU DON'T KNOW,
BEHIND THE MISSION YOU DO.
HIDDEN
FIGURES
3.17
HIDDENFIGURESMOVIE


2×2=4


MONROE



AofA Asymptotics Q&A 1

Q. Give an asymptotic approximation of $e^{H_{2N}-H_N}$ to within $O(\frac{1}{N^2})$

A.

AofA Asymptotics Q&A 1 (improved version)

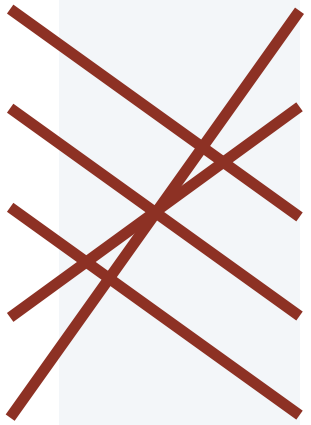
Q. Match each function with an asymptotic expansion.

H_N	$1+\frac{1}{2N}+O(\frac{1}{N^2})$
$\exp(H_{2N}-H_N)-1$	$1+\frac{1}{N}+O(\frac{1}{N^2})$
$\exp(H_N)$	$N+O(1)$
$\exp(\frac{1}{N})$	$1-\frac{1}{N}+O(\frac{1}{N^2})$
$(1+\frac{1}{N})^{-1}$	$1-\frac{1}{2N}+O(\frac{1}{N^2})$
	$\ln N+\gamma+\frac{1}{2N}+O(\frac{1}{N^2})$
	$N+\gamma+O(\frac{1}{N})$

AofA Asymptotics Q&A 2

Q. Match each of the topics described in the book with a mathematician's name.

Approximate a sum with an integral	Laplace
Expand a differentiable function	Ramanujan
Approximate factorials	Euler
Birthday function	Taylor
Approximate a function by swapping tails	Stirling



No, this is not high school, but... *You do not want to appear to be ignorant*!

AofA Asymptotics Q&A 3

Q. Match each expression with an approximation to its value.

1.01^{10}	1.10102
1.05^{10}	1.10462
1.01^{20}	1.22019
1.01^{50}	1.50034
1.01^{100}	1.62889
	1.64463
	2.02300
	2.70481
	2.71828

$$(1+x)^t = \sum_{0\le k\le t} \binom{t}{k} x^k$$

$$= 1 + tx + \frac{t(t-1)}{2}x^2 + O(x^3)$$

$$(1+\frac{1}{N})^t = 1 + \frac{t}{N} + \frac{t(t-1)}{2N^2} + O(\frac{1}{N^3})$$

$$1.01^{10} = 1 + \frac{10}{100} + \frac{90}{20000} + \dots$$

$$\approx 1.1045$$

$$(1+\frac{1}{N})^{\alpha N} = 1 + \frac{\alpha N}{N} + \frac{\alpha^2 N^2}{2N^2} + \dots$$ ✗

$$(1+\frac{1}{N})^{\alpha N} = \exp\big(\alpha N \ln(1+1/N)\big)$$

$$= \exp\big(\alpha N(1/N + O(1/N^2))\big)$$

$$= e^{\alpha} + O(\frac{1}{N})$$

$$1.01^{50} \approx \sqrt{e}$$

AofA Asymptotics Q&A 3

Q. Match each expression with an approximation to its value.