AofA Analytic Combinatorics TEQ 0

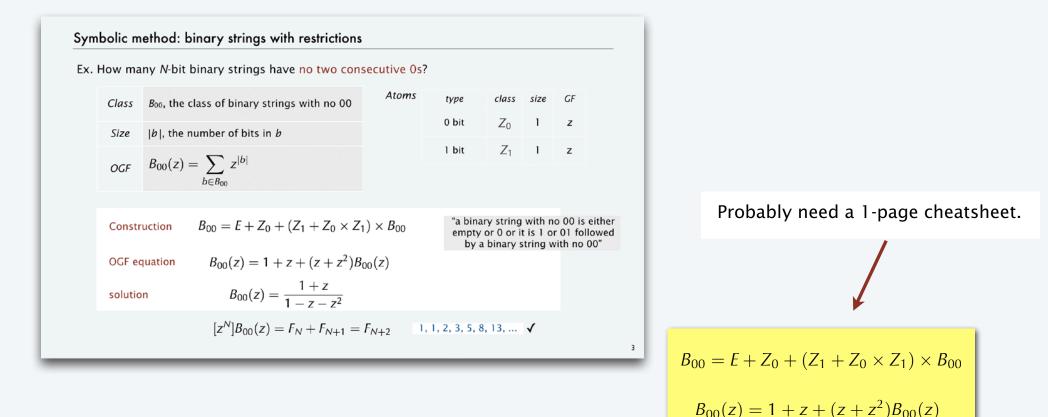
Q. How many binary strings of length *N* have no occurrence of 01?

A. Trick question!

AofA Analytic Combinatorics TEQ 1

Q. How many binary strings of length *N* have no occurrence of 011?

Α.



 $B_{00}(z) = \frac{1+z}{1-z-z^2}$

B011: class of binary strings with no 011

P011: class of binary strings whose only 011 is at the beginning

EQ 1
$$E + (Z_0 + Z_1) \times B_{011} = B_{011} + P_{011}$$

 $1 + 2zB_{011}(z) = B_{011}(z) + P_{011}(z)$ Note: true for any pattern p:
prepending a 0 or a 1 to a p-free binary string
gives a p-free binary string
or a binary string whose only occurrence
of p is at the beginning"EQ 2 $(Z_0 \times Z_1 \times Z_1) \times B_{011} = P_{011}$
 $z^3 B_{011}(z) = P_{011}(z)$ Note: In general, RHS may be more complicated

$$B_{001}(z) = \frac{1}{1 - 2z + z^3} = \frac{1}{(1 - z)(1 - z - z^2)} = \frac{1}{(1 - z)(1 - \phi z)(1 - \phi z)}$$

Probably too intricate for an inclass midterm (borderline, at best)

 $[z^n]B_{001}(z) = \frac{\phi^n}{\sqrt{5}}$

B₀₁₀: class of binary strings with no 010

P010: class of binary strings whose only 010 is at the beginning

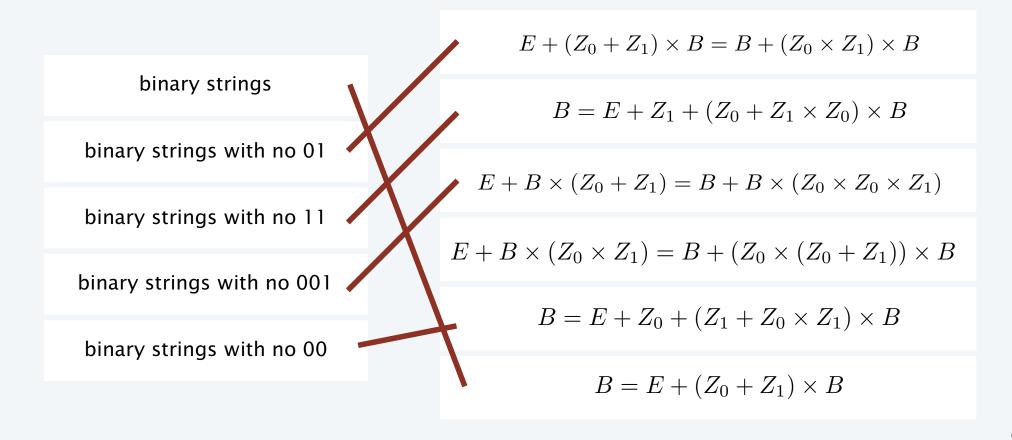
EQ 1
$$E + (Z_0 + Z_1) \times B_{010} = B_{010} + P_{010}$$

EQ 2 $(Z_0 \times Z_1 \times Z_0) \times B_{010} = P_{010} + (Z_0 \times Z_1) \times P_{010}$

if the string from B₀₁₀ begins with 01 constructed string begins with 01010 (two occurrences of 010) 01 followed by a string whose only 010 is at the beginning

AofA Analytic Combinatorics TEQ 1 (improved version)

Q. Match each combinatorial class with a construction.



AofA Analytic Combinatorics TEQ 2 (2015 exam)

Q6. Let
$$k_n = [z^n] \frac{1}{\sqrt{1-3z}} \log \frac{1}{1-2z}$$
. Then:
O $k_n \sim \log 3 \frac{3^n}{\sqrt{\pi n}}$
O $k_n \sim \sqrt{2} \frac{2^n}{n}$

Lesson: Need a 1-page cheatsheet.

with correct version of formula!

$$[z^{n}] \frac{f(z)}{(1-z/\rho)^{\alpha}} \sim \frac{f(\rho)}{\Gamma(\alpha)} \rho^{-n} n^{\alpha-1}$$

$$\Gamma(\frac{1}{2}) = \sqrt{\pi}$$

$$\Gamma(1) = 1$$

$$\Gamma(s+1) = s\Gamma(s)$$

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AofA Analytic Combinatorics TEQ 2 (improved version)

Q. Match each expression with an approximate value

Γ

 $\Gamma(s -$

$$\begin{bmatrix} z^{n} \end{bmatrix} \frac{f(z)}{(1-z/\rho)^{\alpha}} \sim \frac{f(\rho)}{\Gamma(\alpha)} \rho^{-n} n^{\alpha-1} \\ \Gamma(\frac{1}{2}) = \sqrt{\pi} \\ \Gamma(1) = 1 \\ \Gamma(s+1) = s\Gamma(s) \end{bmatrix} \begin{bmatrix} z^{n} \end{bmatrix} \frac{1}{(1-3z)} \ln \frac{1}{1-2z} \\ \begin{bmatrix} z^{n} \end{bmatrix} \frac{1}{(1-2z)(1-3z)} \\ \begin{bmatrix} z^{n} \end{bmatrix} \ln \frac{1}{\sqrt{1-2z}} \\ \begin{bmatrix} z^{n} \end{bmatrix} \ln \frac{1}{1-2z} \\ \begin{bmatrix} z^{n} \end{bmatrix} \frac{1}{1-3z} \ln \frac{1}{1-2z} \\ \end{bmatrix} \begin{bmatrix} x^{n} \ln 3 \\ \frac{3^{n}}{\sqrt{\pi n}} \\ \frac{3^{n} \ln 3}{\sqrt{\pi n}} \\ \end{bmatrix}$$

 2^{n-1}

The hard way:

$$[z^{n}]\frac{1}{(1-2z)(1-3z)} = [z^{n}]\left(\sum_{n\geq 0} 2^{n}z^{n}\right)\left(\sum_{n\geq 0} 3^{n}z^{n}\right)$$
$$= \sum_{0\leq k\leq n} 2^{k}3^{n-k}$$
$$= 3^{n}\sum_{0\leq k\leq n} (2/3)^{k}$$

Still not convinced?

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Input interpret	ation:				
	1	point z	= 0		
series	$\overline{(1-2z)(1-3z)}$	order z	10		
					Open code 🔿
Series expans	ion at z=0:				
	$19 z^2 + 65 z^3 + 211 z^4$				More terms
$6305 z^7$	$+ 19171 z^{8} + 58025$				More terms
6305 z ⁷ (Taylor serie	$+ 19171 z^8 + 58025$ es) when $3 z < 1$		$^{10} + O(z^{11})$	rs — bc — 70×10	More terms