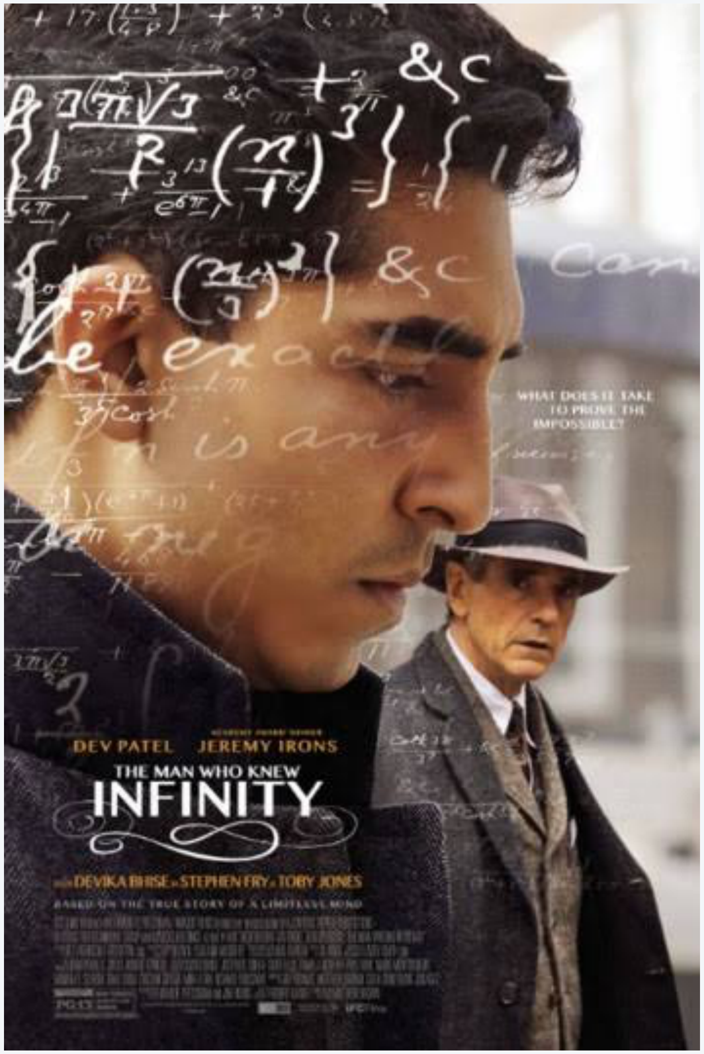
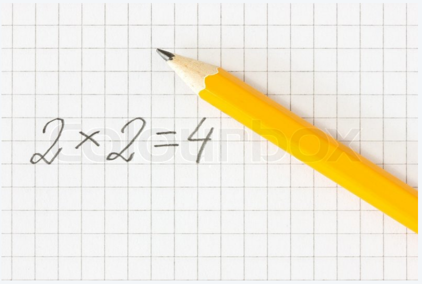


" An equation means nothing to me unless it expresses a thought of God."

– Srinivasa Ramanujan

$$Q(N) \equiv \sum_{1 \le k \le N} \frac{N!}{(N-k)!N^k} = \sqrt{\frac{\pi N}{2}} + O(1)$$


BASED ON THE UNTOLD TRUE STORY
MEET THE WOMEN YOU DON'T KNOW, BEHIND THE MISSION YOU DO.
HIDDEN FIGURES
3.17
HIDDENFIGURESMOVIE


2×2=4


MONROE

AofA Asymptotics TEQ 1

Q. Give an asymptotic approximation of $e^{H_{2N}-H_N}$ to within $O(\frac{1}{N^2})$

A.

AofA Asymptotics TEQ 1 (improved version)

Q. Match each function with an asymptotic expansion.

Function
H_N
$\exp\big(H_{2N} - H_N\big) - 1$
$\exp\big(H_N\big)$
$\exp(\frac{1}{N})$
$(1 + \frac{1}{N})^{-1}$

Asymptotic expansion
$1 + \frac{1}{2N} + O(\frac{1}{N^2})$
$1 + \frac{1}{N} + O(\frac{1}{N^2})$
$N + O(1)$
$1 - \frac{1}{N} + O(\frac{1}{N^2})$
$1 - \frac{1}{2N} + O(\frac{1}{N^2})$
$\ln N + \gamma + \frac{1}{2N} + O(\frac{1}{N^2})$
$N + \gamma + O(\frac{1}{N})$

AofA Asymptotics TEQ 2

Q. Match each of the topics described in the book with a mathematician's name.

Approximate a sum with an integral	Laplace
Expand a differentiable function	Ramanujan
Approximate factorials	Euler
Birthday function	Taylor
Approximate a function by swapping tails	Stirling

No, this is not high school, but... *You do not want to appear to be ignorant*!

AofA Asymptotics TEQ 3

Q. Match each expression with an approximation to its value.

1.01^{10}
1.05^{10}
1.01^{20}
1.01^{50}
1.01^{100}

1.10102
1.10462
1.22019
1.50034
1.62889
1.64463
2.02300
2.70481
2.71828

$$(1+x)^t = \sum_{0 \le k \le t} \binom{t}{k} x^k$$

$$= 1 + tx + \frac{t(t-1)}{2} x^2 + O(x^3)$$

$$(1+\frac{1}{N})^t = 1 + \frac{t}{N} + \frac{t(t-1)}{2N^2} + O\Big(\frac{1}{N^3}\Big)$$

$$1.01^{10} = 1 + \frac{10}{100} + \frac{90}{20000} + \ldots$$

$$\approx 1.1045$$

$$(1+\frac{1}{N})^{\alpha N} = 1 + \frac{\alpha N}{N} + \frac{\alpha^2 N^2}{2N^2} + \ldots$$

$$(1+\frac{1}{N})^{\alpha N} = \exp\big(\alpha N \ln(1+1/N)\big)$$

$$= \exp\big(\alpha N (1/N + O(1/N^2))\big)$$

$$= e^{\alpha} + O\big(\frac{1}{N}\big)$$

$$1.01^{50} \approx \sqrt{e}$$

AofA Asymptotics TEQ 3

Q. Match each expression with an approximation to its value.

1.01^{10}	1.10102
1.05^{10}	1.10462
1.01^{20}	1.22019
1.01^{50}	1.50034
1.01^{100}	1.62889
	1.64463
	2.02300
	2.70481
	2.71828