# AofA Recurrences Q&A 1

Q. Solve the recurrence

$$na_n = (n-3)a_{n-1} + n \quad \text{for} \quad n \geq 3 \quad \text{with} \quad a_n = 0 \quad \text{for} \quad n \leq 2$$

| $n$ | $a_n$ |
|---|---|
| 3 | 1 |
| 4 | 5/4 |
| 5 | 3/2 |
| 6 | 7/4 |

A.

summation factor

$$\overline{n(n-1)(n-2)}a_n = \overline{(n-1)(n-2)}(n-3)a_{n-1} + \overline{n(n-1)(n-2)} \quad \text{for} \quad n \geq 3$$

$$\binom{n}{3}a_n = \binom{n-1}{3}a_{n-1} + \binom{n}{3} \quad \text{for} \quad n \geq 3$$

$$= \sum_{3 \leq k \leq n} \binom{k}{3} \quad = \binom{n+1}{4}$$

$$a_n = \frac{n+1}{4}$$

Note. We try hard to avoid answers that depend on detailed calculations.

# AofA Recurrences Q&A 1 (rejected version)

Q. Solve the recurrence

$$na_n = (n-3)a_{n-1} + n \quad \text{for} \quad n \geq 4 \quad \text{with} \quad a_n = 0 \quad \text{for} \quad n \leq 3$$

A.

$$n(n-1)(n-2)a_n = (n-1)(n-2)(n-3)a_{n-1} + n(n-1)(n-2) \quad \text{for} \quad n \geq 4$$

$$\binom{n}{3}a_n = \binom{n-1}{3}a_{n-1} + \binom{n}{3} \quad \text{for} \quad n \geq 4$$

$$= \sum_{4 \leq k \leq n} \binom{k}{3} = \binom{n+1}{4} - 1$$

$$a_n = \frac{n+1}{4} - \frac{1}{\binom{n}{3}}$$

| $n$ | $a_n$ |
|---|---|
| 4 | 1 |
| 5 | 7/5 |
| 6 | 17/10 |
| 7 | |

Too complicated for an inclass exam? Probably.

## AofA Recurrences Q&A 2

Q. Which of the following is true of the number of compares used by Mergesort ?

$$C_N = C_{\lfloor N/2 \rfloor} + C_{\lceil N/2 \rceil} + N \quad \text{for} \quad N \geq 2 \quad \text{with} \quad C_1 = 0$$

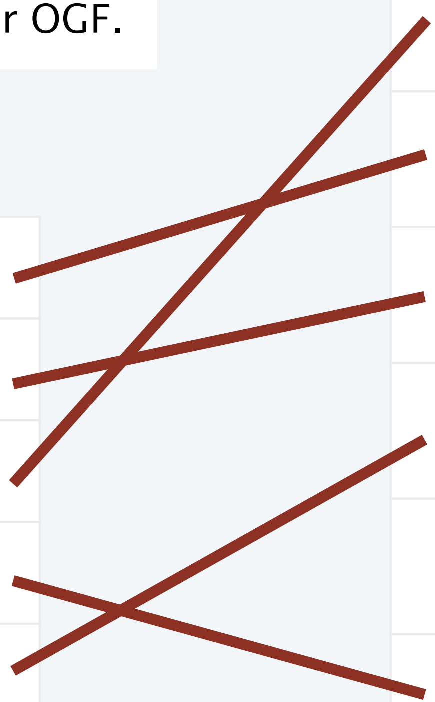
| | |
|---|---|
| Order of growth is $N \lg N$ | **T** |
| Exactly $N \lg N$ when $N$ is a power of 2 | **T** |
| Is equal to the number of 1s in the binary representation of the numbers $< N$ | **F** |
| Has periodic behavior | **T** |
| Is less than $N \lg N + N/4$ for all $N$ | **T** |

Some questions are of the form: *Did you watch the lectures and/or do the reading*?

# AofA GFs Q&A 1

Q. Match each of the following sequences with their OGF.

| |
|---|
| 0, 0, 1, 3, 6, 10, ... |
| 0, 0, 1/2, 0, 1/4, 0, 1/6, ... |
| 1, 3, 9, 27, 81, 243, ... |
| 1, 1 + 1/2, 1 + 1/2 + 1/3, ... |
| 3, 3, 3, 3, 3, ... |



| |
|---|
| $\frac{1}{1-3z}$ |
| $\frac{z^2}{(1-z)^3}$ |
| $\ln\frac{1}{1-z^2}$ |
| $\frac{3}{1-z}$ |
| $\ln\frac{1}{1-2z}$ |
| $\frac{1}{1-z}\ln\frac{1}{1-z}$ |
| $\frac{1}{(1-z)^3}$ |

## AofA GFs Q&A 2

Q. Suppose that $a_n$ satisfies $a_n = 9a_{n-1} - 20a_{n-2}$ for $n > 1$ with $a_0 = 0$ and $a_1 = 1$

What is $\lim_{n\to\infty} a_n/a_{n+1}$ ?

A. **5**

$$a(z) = \frac{z}{1 - 9z + 20z^2} = \frac{z}{(1-4z)(1-5z)} = \frac{1}{1-5z} - \frac{1}{1-4z}$$

$$a_n = 5^n - 4^n$$

# AofA GFs Q&A 2 (improved form)

Q. Suppose that $a_n$ satisfies $a_n = 9a_{n-1} - 20a_{n-2}$ for $n > 1$ with $a_0 = 0$ and $a_1 = 1$

Fill the box corresponding to the value of $\lim_{n\to\infty} a_n/a_{n+1}$ *and* justify your answer.

○ 3 ○ 4 ● 5

Justification must be 1-3 lines

$$a(z) = \frac{z}{1 - 9z + 20z^2} = \frac{z}{(1-4z)(1-5z)} = \frac{1}{1-5z} - \frac{1}{1-4z}$$

$$a_n = 5^n - 4^n$$

No credit for wrong or unjustified answers.

# AofA GFs Q&A 3

Q. Fill the circle corresponding to the value of

$$[z^n] \sum_{0\le k\le n} \binom{2k}{k}\binom{2n-2k}{n-k}$$

*and* justify your answer.

○ $2^n$ ● $4^n$ ○ $2^{n/2}$

It is $[z^n]\Big(\dfrac{1}{\sqrt{1-4z}}\Big)^2$